Theory of Automata and Computation

Input → Automatic Conversion → Output

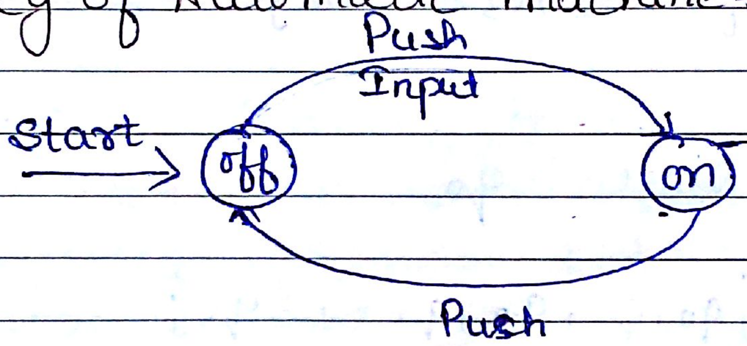
Language

a, b

Grammar Rules

1. b after a
2. Starting Symbol a

→ Eg of Automatic machine :-

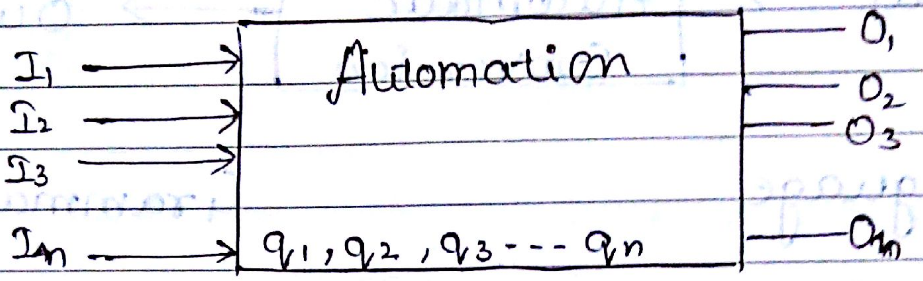


→ State
valid state → on or off
Input → Push
Output :→ Change of State

Defination of Automation:-
An Automation is defined as a System where energy, materials and information are transffered transformed and used for performing some functions without direct participation of man eg are Automatic machine Tools, Automatic washing machine

1) All finite States of an Automation are represented by a circle
2) Input of the System are represented by arc. It Shows the external influence on the System
3) One of the State is designated the Start state

5) The State in which the system is placed initally
6) It is indicate by the word Start and an arrow leading to that State



Model of Discrete Automation

alphabet ↑

Denote by Σ ① Input → $\{I_1, I_2, I_3, I_4, --- I_n\}$ → finite

Q ② Valid State → $\{q_1, q_2, q_3 --- q_n\}$

Denoted by F ③ Output → $\{O_1, O_2, O_3 --- O_n\}$ → limitted
↓

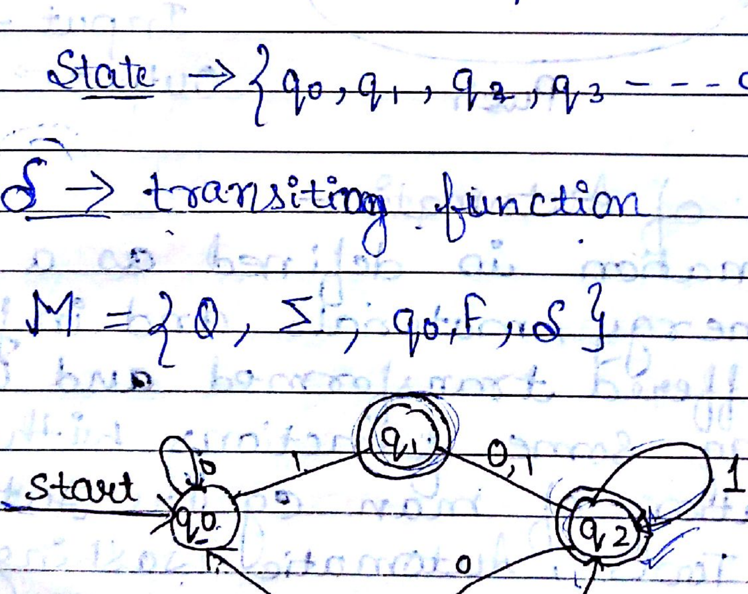
Final state

④ Initial state → q_0

State → $\{q_0, q_1, q_2, q_3 --- q_4\}$

⑤ δ → transiting function

$M = \{Q, \Sigma, q_0, F, \delta\}$



0 1 0 1 1

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

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Central Concepts of Automata Theory :

1) Alphabets (set of symbols) $\Sigma = \{0, 1\}$
2) Strings (list of symbols from an alphabet)
3) Language (strings from same alphabet)

This concepts include alphabets, strings & Language

1) **Alphabet:** It is a finite non empty set of symbol denoted by Σ eg $\Sigma = \{0, 1\}$ it is eg of Binary set
eg $\Sigma = \{a, b, c, z\}$ it is a set of lower case letter

2) **String:** A string is a finite sequence of symbols choosen from some alphabet (Σ) sigma
eg 01101 is a stringe from the Binary alphabet $\Sigma = \{0, \}$

- **Empty string:** It is defined as a string with zero occurances of symbols. This string denoted as ϵ is a string that may be choosen from any alphabet.

$$q_0 \xrightarrow{\epsilon} q_1$$

- **Length of a string:** String are classified by there length. Length means no of position for symbol in a string. The standard notation for the length of string is donated as w

$w = \{1100011\}$

$|w| = 7$ $\quad |\epsilon| = 0$

The Length of String $|w|$ is 7.

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- **Powers of an alphabet (Σ):** If Σ is a alphabet we can express the set of all string of a certain length from that alphabet by using an exponential notation
Σ^k where K is an integer where K = 3
eg: $\Sigma^3 = \{000, 001, 101, 010, 110, \text{---} 111\}$

$$\Sigma^0 = \{\epsilon\} \quad , \quad \Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \text{---} \Sigma^n$$

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3$$

- **Contactination of String:**
$x = 01101, \; y = 0011 \longrightarrow xy = 011010011$

3) **Languages:** A set of strings all of which are choosen from some Σ^*, where Σ is a particular alphabet is called language
If L is a language and Σ is alphabets and language is a subset of alphabets
$L \subset \Sigma^*$ over Σ
$L = \{\Sigma'$
Any language over Σ is also the language of any alphabets i.e supper set of Σ

- **Why we are using term language:**

Because every language can be viewed as set of string.

eg: $\Sigma = \{a, b, c, \text{----} z\}$

L is English $L \subseteq \Sigma^*$

English is collection of legal english word is a set of string over the alphabets that consists of all the 26 letter

C-language+ legal program are subset of the possible string that can be formed from the alphabets of the language
This alphabet is subset of ASCII

$\phi \rightarrow$ Empty language is a language of over any alphabet
$\{\epsilon\} \rightarrow$ A language consisting of only the empty string

$$\phi \neq \{\epsilon\}$$

ϕ has no string $\{\epsilon\}$ has 1 string

NOTE: For any language alphabet are finite. A language can have an infinite no of string, are restricted to consist of string from one fixed finite alphabet

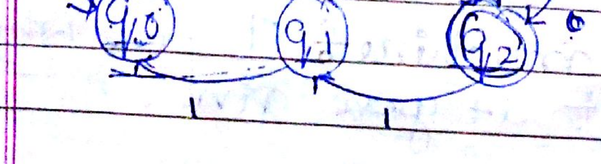
Assignment: Transition System

FSA / DFSA / DFA :

$$M = \{Q, \Sigma, \delta, q_0, F\}$$

$$M = \{H, \Sigma, \delta, H_0, F\}$$

Transition Diagram



$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{0, 1\}$
$\delta = Q \times \Sigma \rightarrow Q$
Intinal State q_0
$F = \{q_2\}$

$\delta \rightarrow Q \times \Sigma$

Transition Table:

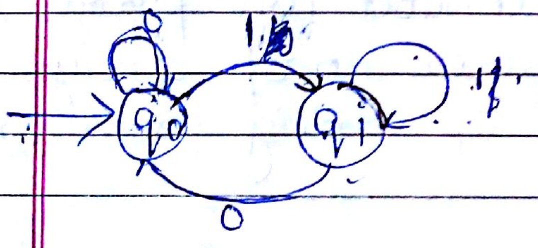
State	Input 0	Input 1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
q_2^* (q_2)	q_2	q_1

Final State

eg:

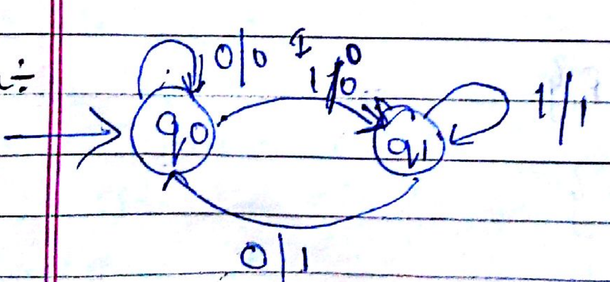
State	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_0	q_1

Diagram:

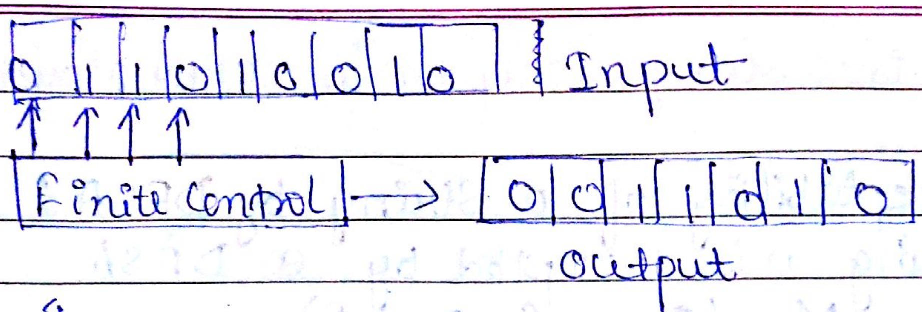


eg → Input → 01101001
Output → 00110100

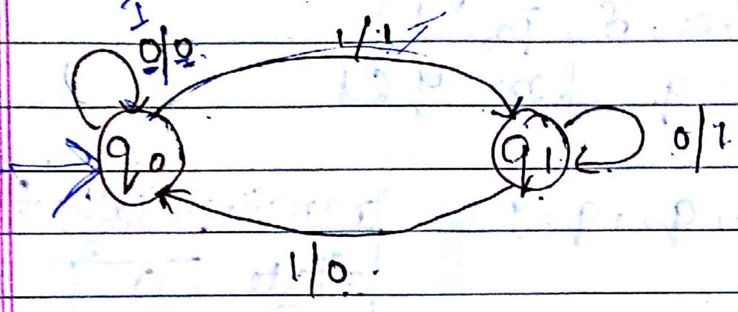
Given:



One moment delay machine: Input which it take the same output it ~~gave~~ give.
It start from zero



eg:



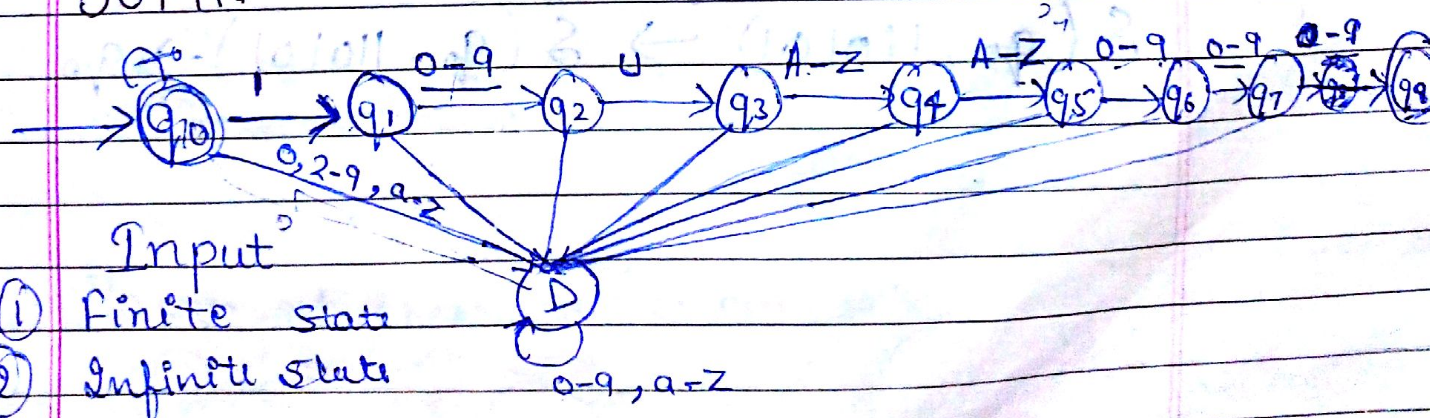
State	0		1	
	→	output	State	Output
q0	q0	0	q1	1
q1	q1	1	q0	0

Input → 1 1 0 0 1 1 0 1
Output → 1 0 0 0 1 0 0 1

If odd no of ones in Input then output of Last is one i.e is called parity checker and viceversa for even no of ones in Input then output of last is Zero

11UCS001

BUPIN



Input
① Finite State
② Infinite State

$\Sigma = \{0-9, a-z\}$

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It is a string acceptor or BUPIN acceptor

Acceptability of a String by DFSA :- A String w is accepted by a DFSA

$M = (Q, \Sigma, \delta, q_0, F)$

if $\delta(q_0, w) = q$ for $q \in F$

eg $Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$F = \{q_0\}$

Transition table :-

State	Σ 0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Input - 110101

$\hat{\delta} = \delta(q_0, 110101) \rightarrow \delta(q_1, 10101) \rightarrow$

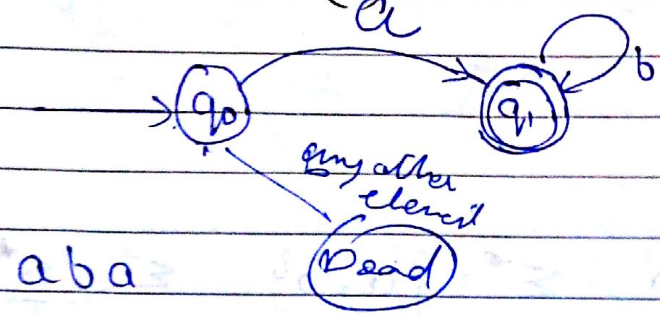
$\delta(q_0, 0101) \rightarrow \delta(q_2, 101) \rightarrow \delta(q_3, 01)$

$\delta(q_1, 1) = q_0 \in F$

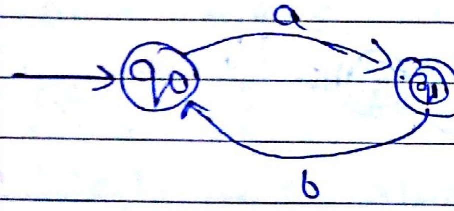
Hence this machine accept the String

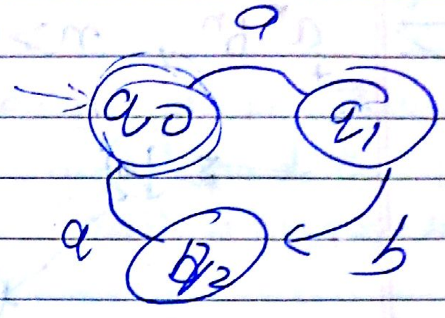
$\delta(q_0, 110101) \rightarrow \hat{\delta}(q_0, 110101) \rightarrow q_0$

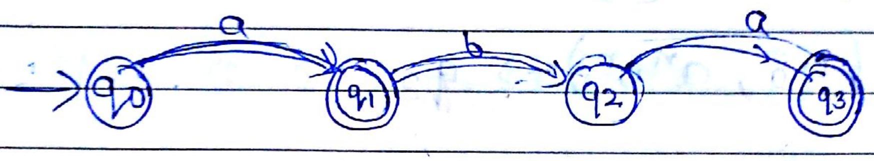
Eg: $ab^n \rightarrow a(b)^*$



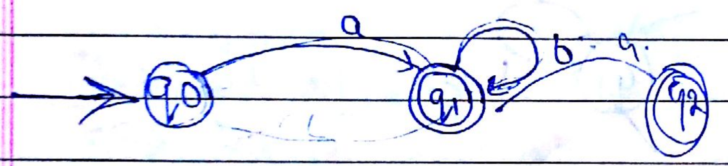
→ aba





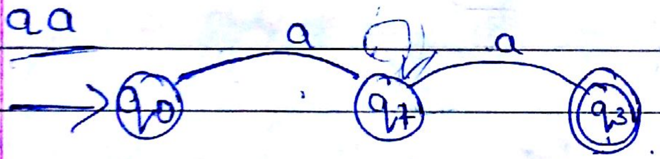


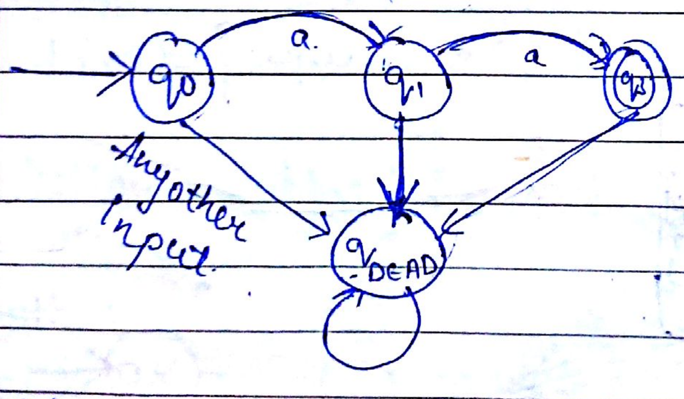
→ $ab^n a \rightarrow a(b)^* a$.



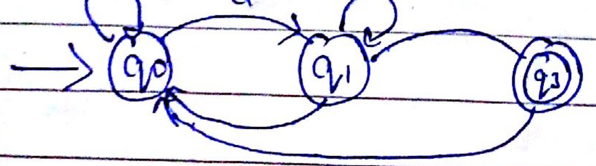
→ aa

$\varepsilon \{a, b\} = aa$

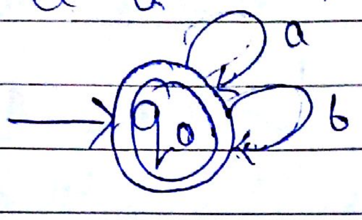




To prevent q_{DEAD} we use try catch

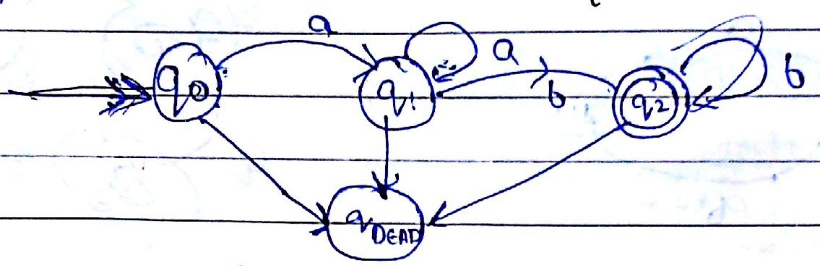


→ $a^* b^* \longrightarrow 0^* 1^*$



→ $a^n b^n$, $n > 0$ $\Sigma^* = \{a, b\}$ Σ^+, $\Sigma^* \to 0$ occurrence included

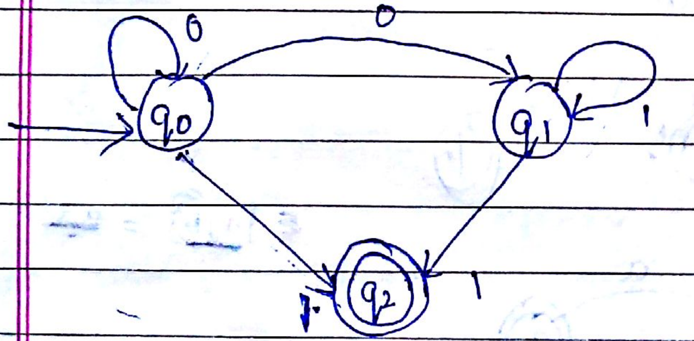
Σ^+



a-b

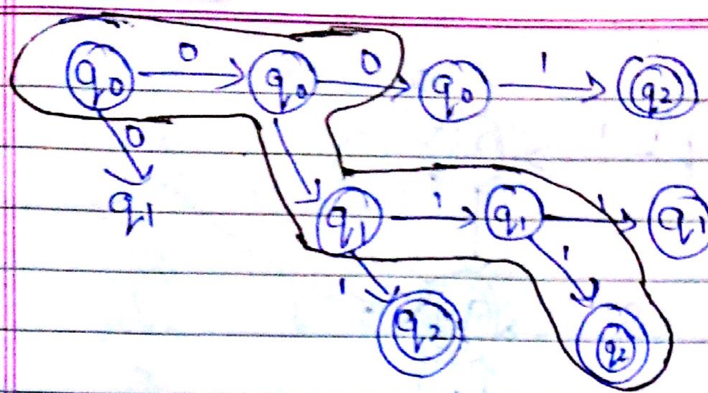
$\delta(q_0, a^n b^n) \to q_2$

NFSM (Non Deterministic finite State Machine):



Transition Table:

State	Σ: 0	1
→ q_0	$\{q_0, q_1\}$	q_2 $\cancel{\{q_1, q_2\}}$
q_1	ϕ	$\{q_1, q_2\}$
q_2	ϕ	ϕ



0011

Some moves of the machine cannot be determine uniquely by the input symbol and the present state such machine are called Non Deterministic Finite automation

$$\underbrace{M_N}_{\downarrow} = \{Q, \Sigma, \delta, q_0, F\}$$

Non Deterministic machine

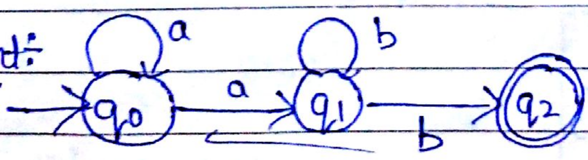
$Q \rightarrow$ Set of state
$\Sigma \rightarrow$ input alphabet
$\delta \rightarrow Q \times \Sigma \rightarrow 2^Q$ $p(Q)$

$$L(M) = \{w \mid w \in \Sigma^*\}$$

$$L(M) = \{w \mid w \in \Sigma^*, \delta(q_0, w) \rightarrow Q \cap F \neq \phi\}$$

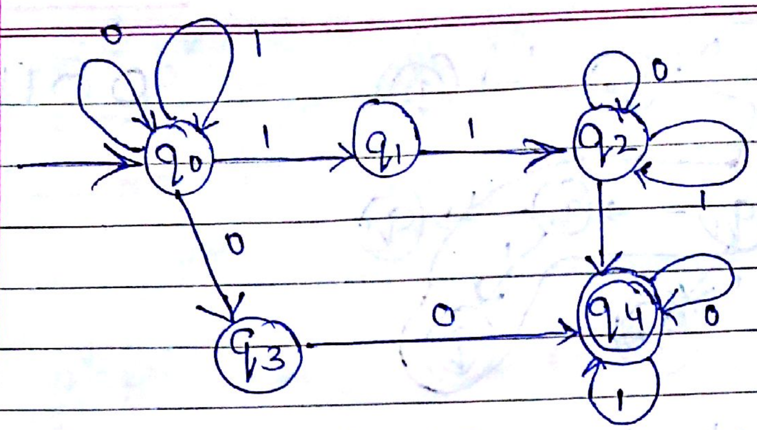
$\rightarrow$ Difference b/w DFA & NDFA

Assignment:



Input $\rightarrow$ aaaabb

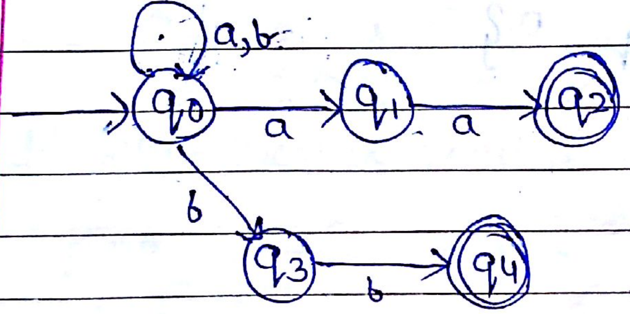
Que →



Input - 0100

$q_0 \rightarrow q_0 \rightarrow q_0 \rightarrow q_3 \rightarrow q_4$

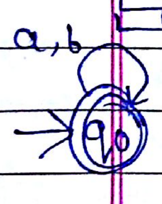
Que →

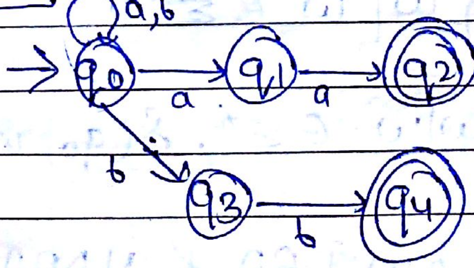


→ NDFA

$a^* bb$, $b^* aa$

$(a+b)^* . (bb+aa)$

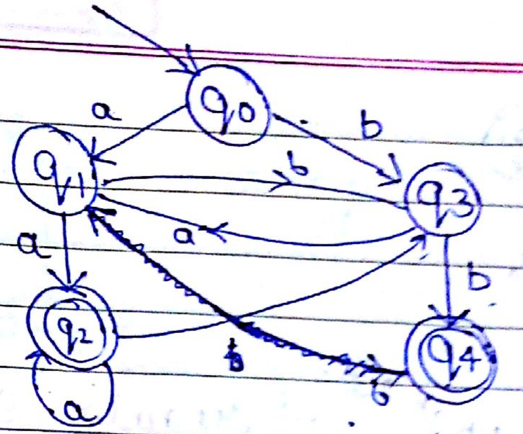




Transition table :-

State	a	b
→ q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	q_2	ϕ
q_2	ϕ	ϕ
q_3	ϕ	q_4
q_4	ϕ	ϕ

3



$(a+b)^*(aa+bb)$

aaabb
bbbaa

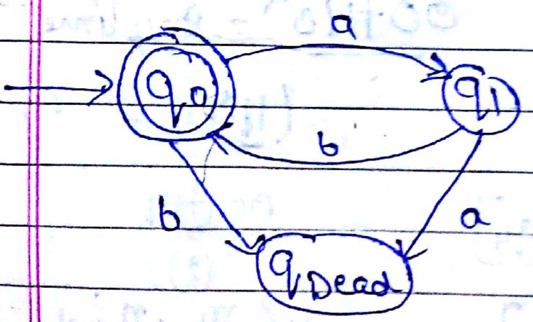
→ To Convert NDFA and DFA.

NDFA

State	0	1
q_0	$\{q_0, q_1\}$	ϕ
q_1	ϕ	$\{q_1, q_2\}$
q_2	ϕ	ϕ

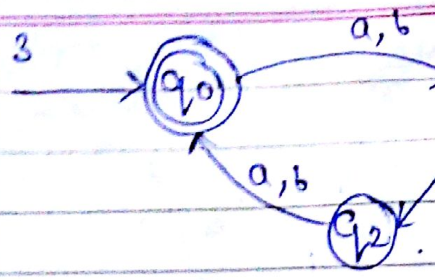
DFA

q_0	$[q_0, q_1]$	ϕ
$[q_0, q_1]$	$[q_0\, q_1]$	$[q_1, q_2]$
$[q_1\, q_2]$	ϕ	$[q_1\, q_2]$
ϕ	ϕ	ϕ



$(ab)^*$ abababab.

→



$\Sigma = \{a,b\}$

$\Sigma^3 =$ aaa, bbb, abb, baa , $(a+b)^n$ $n*3$ → regular Expression

→ NFA ↔ DFA

Transition table ↔ Regular Expression

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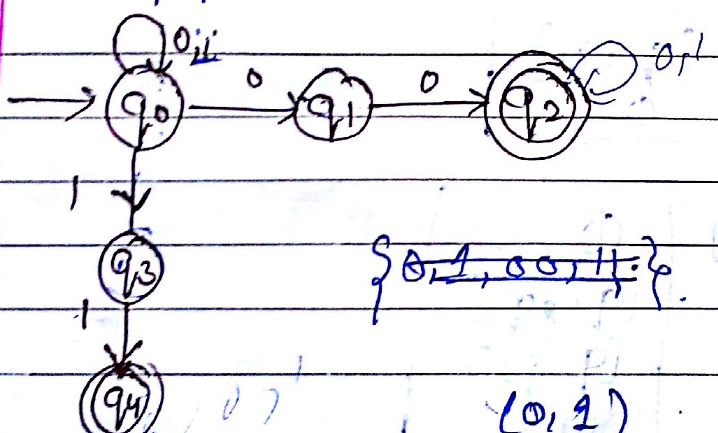
↳ Theory of Automata Notes

eg:



$(0+1)^*(00+11)$.

$\{\cancel{0,1,00,11}\}$.

$\Sigma = \{0,1\}$

$(0,1)$. 0^2 1^2 $\Sigma^*(0^m, 1^n$ where $m \geq 1$, $n \geq 1)$

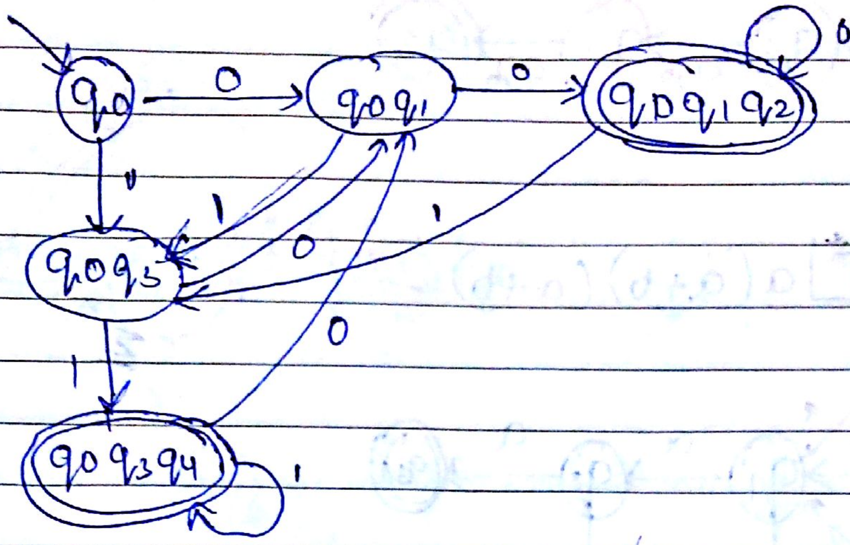
$00+11\,0^*$ = Any time

(11^*). 11 α.

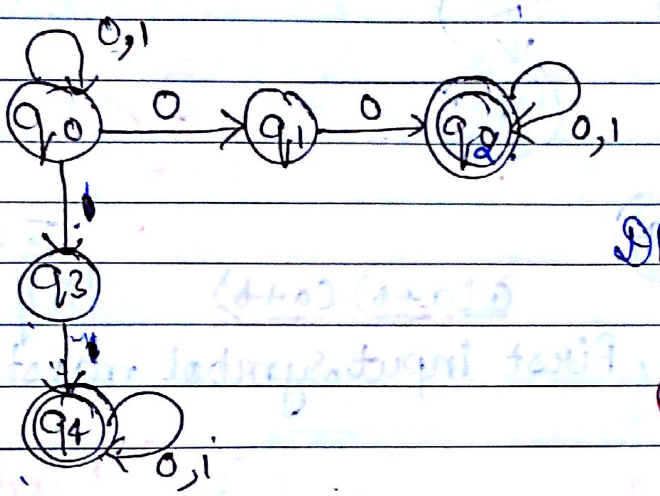
$00 + 11$

$(0^m, 1^n$ where $m \geq 1$, $n \geq 1)$.

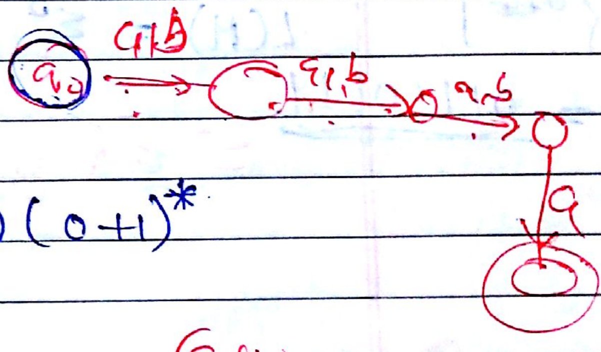
State	0	1
→q_0	$[q_0 q_1]$	$[q_0, q_3]$
$[q_0 q_1]$	$[q_0 q_1 q_2]$	$[q_0, q_3]$
$[q_0 q_3]$	$[q_0, q_1]$	$[q_0, q_3 q_4]$
$[q_0 q_1 q_2]^*$	$[q_0 q_1 q_2]$	$[q_0\ q_3]$
$[q_0 q_3 q_4]^*$	$[q_0, q_1]$	$[q_0, q_3, q_4]$



eg:-

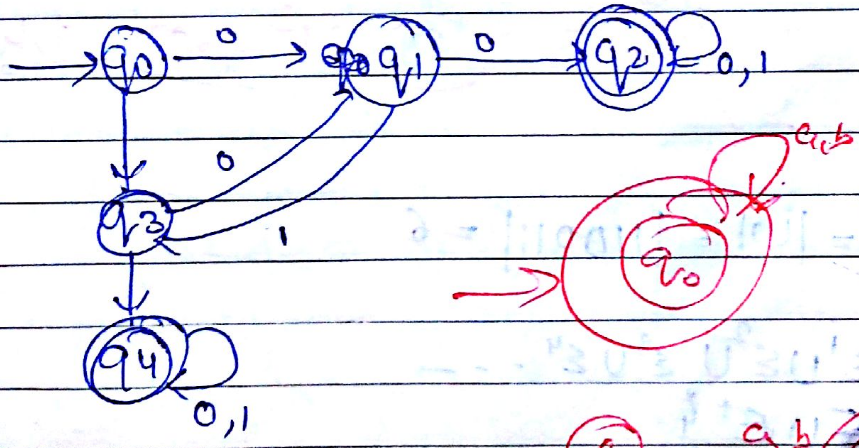


DFA or NDFA regular Expres:



Expression $(0+1)^*$, $(00+11)(0+1)^*$

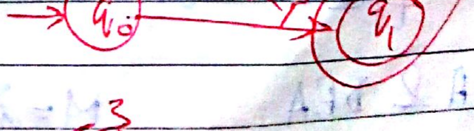
$(a+b)$



$(a+b)^*$

$(a+b)^+$

a b

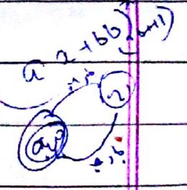


$\Sigma = \{a, b\}$

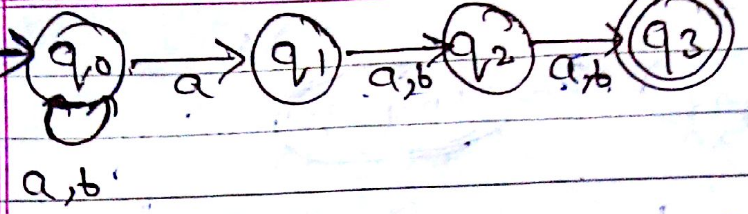
Σ^3

Σ^4

a

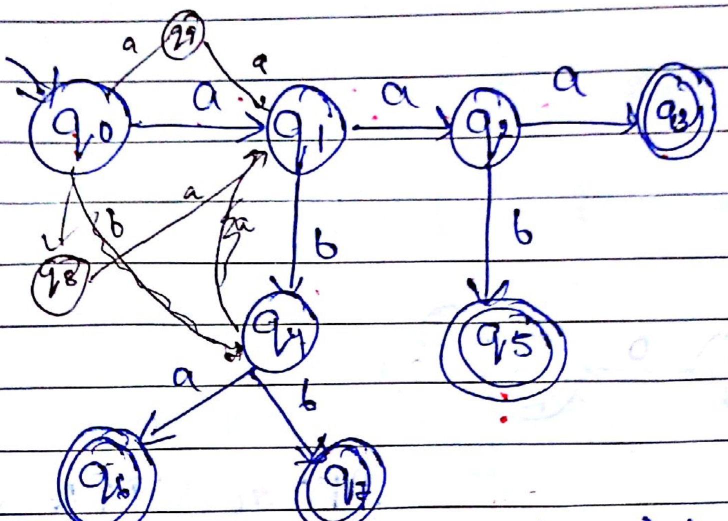


Eg:



$(a+b)^* \, a(a+b)(a+b)$ infinite

Σ^*



aaa
aab
abb

$\Sigma = \{a, (a,b)\}$

$a(a+b)(a+b)$ $|W| = 3.$

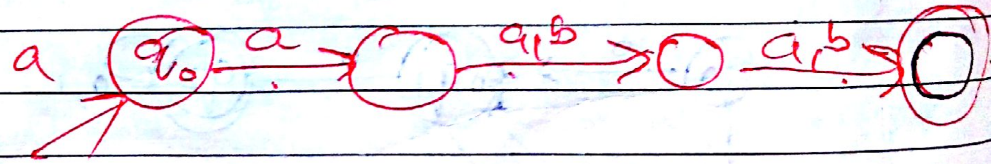
$\{0,1\} = \Sigma$

$L(M) = \Sigma^3$, first input symbol must be a

$= 0111011$

$\Sigma = \{a, b\}$ $\Sigma^3 = \{aaa, bbb, aba,$

$\Sigma^n.$



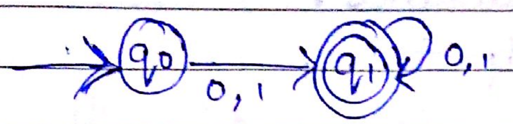
→ $110011 = |W| = |110011| = 6$

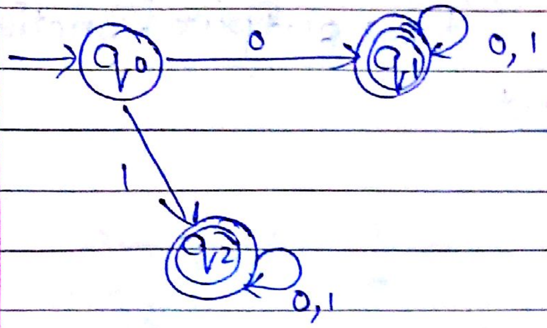
→ $\Sigma^+ \rightarrow \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \Sigma^4$ ---

$\Sigma^* \{ \varepsilon \cup \Sigma^+ \}$

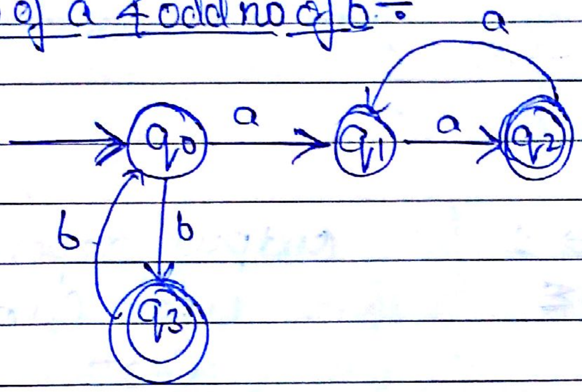
NFA & DFA. $M = \{Q, \Sigma, \delta, q_0, F\}$

$\delta = \Sigma \times Q = 2^Q.$

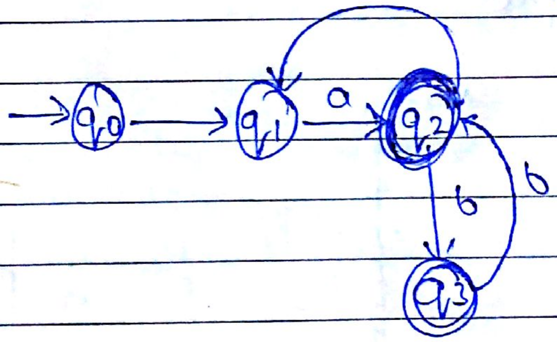




<u>even no of a & odd no of b :</u>



OR



Mealy and Moore Machines :-

Automata with output

6 Tuple Machine

$M = \{\Sigma, Q, \delta, q_0, F, \underbrace{\lambda}_{\text{output function}}\}$ → output function.

	0	1	Output
q_0	q_1	q_2	1

$\lambda(q_0, 0) \rightarrow 1$

$\lambda \rightarrow \Sigma \times Q \rightarrow \Sigma$

→ Moore Machine :-

$\lambda(q_0) \rightarrow 1$

$\lambda \rightarrow Q \rightarrow \Sigma$

output depent upon Current State

	0	1	output
q_0	q_1	q_2	1

→ Mealy Machine :-

	0	o/p	1	o/p		0	output
q_0	q_0	1	q_1	0	~~q_0~~	q_1	1 q_2

Moore

output depent upon Input symbol & Current State

Q Draw a Moore Machine, States are q_0, q_1, q_2, q_3 Convert into Mealy Machine

	0	1	output
q_0	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Sol

~~q_0~~	0	λ	1	λ
q_0	q_3	0	q_1	1
q_1	q_1	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Mealy Machine

Que

Convert Mealy machine to Moore machine

	0		1	
	state	output	state	output
q_1	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

	0	1	output
q_1	q_3	q_2	0
q_3	q_2	q_1	1
q_{20}	q_1	q_4	1
q_{21}	q_1	q_4	0
q_{40}	q_4	q_3	1
q_{41}	q_4	q_4	0

$\Sigma = \{0, 1\}$

Zero $\leftarrow (0+1)^*$ $\quad (0+1)\,(0+1)\,(0+1)$ $\quad \Sigma = \{0, 1\}$.

$(0+1)^+ \rightarrow$ for length - 1, 2.

→ Any String ending with two Zero

$(0+1)^*\, 00$

→ String containing at least two zero

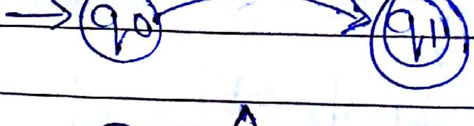
$(0+1)^*\; 0\,(0+1)^*\; 0\,(0+1)^*$

→ $a^n b^m c^k$ where $\begin{Bmatrix} m \geq 0 \\ n \geq 0 \\ k \geq 0. \end{Bmatrix}$

$a^* b^* c^*$

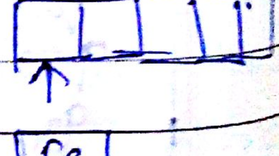
Regular Expression:

ii) $\Sigma^* L$

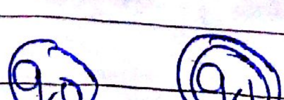


empty ~~set~~ String

NDFA with Null moves or ε moves



1) For ϕ. →



empty set